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A Study On Connections And Vector Fields In Kaehlerian Spaces

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Abstract

In the Levi-Civita connection of N, a vector field is considered to be concircular if and only if every vector X is tangent to ∇Xv = µX. Assuming that X is a function such that v is not constant, then a nontrivial concircular vector field is one that fulfills the equation ∇Xv = µX for all X. Small holomorphic projective transformations on compact Kaehlerian manifolds have been developed and computed by Izumi [2] and Kazanari. Projective equivalentes of holomorphic ammeters have been explored, as shown by Malave Guzman [3]. Pseudo-analytic vectors on pseudo-Kaehlerian manifolds were one of the many open areas that Negi[5] explored and pondered. A new analytic HP-transformation designed for use in predominantly Kaehlerian domains was recently published and developed by Negi et al. [6]. As a result of this research, we are able to determine the holomorphic properties of Einsteinian and constant curvature manifolds, as well as the dimensions and dimensions of a Kahlerianmani fold associated with recurrent curvature killing vector fields in H-projective spacetimes. With the geometric features of the calculated harmonic and scalar curvatures, we can produce overwhelming vectorial fields for nearly complex Kaehlerian holomorphically projective recurrent curvature manifolds.

Keyword

Vector, Kaehlerian, Space, Symmetric

Introduction

one that is n-dimensional in Kaehlerian space and has a constant holomorphic sectional curvature In mathematics, the letter Mn stands for a complicated space time shape such as c. (c). For c > 0, = 0, and = 0, it is proved that complex space is isometric to complex Euclidean space Cn, complex hyperbolic space HnC, and complex projective space PnC.

A. Fialkow defined concircular vector fields on Riemannian manifolds, and below are various circumstances under which A holds:

(1.1) ∇Xv = μX

When N is defined by the Levi-Civita connection and X is the tangent vector to N, then denotes a non-trivial function on N. Non-triviality in a concentric vector field is defined by the presence of a function that is not constant (1.1). Similarly, on pseudo-Riemannian manifolds, one can provide precise definitions of concircular vector fields.

related in a direct, continuous family tree [7] The torsion tensor T of an isotropic link ∇˜ on a differentiable n-dimensional manifold looks like this.

$$\begin{aligned} T(X,Y) &= \tilde{\nabla}_X Y - \tilde{\nabla}_Y X - [X,Y] \\ &= \eta(Y)\phi X - \eta(X)\phi Y, \end{aligned}$$

where is a tensor of rank 1 form (1, 1). In fact, in the simplest situation, X = X [6,] the quarter-symmetric connection can be written as a semi-symmetric one. This article introduces a quarter-symmetric connection, which can be used to increase the semi-symmetric connection's ∇˜ scope of use. If this condition holds, then the linear connection is quarter-symmetric.

$$(\tilde{\nabla}_X g)(Y,Z) = 0,$$

We show that the Lie algebra of vector fields on M is X (M), where M is a manifold, for any X, Y, and Z in X (M) (M).

If an n-dimensional differentiable manifold M has a tensor field of type (1, 1), a vector field, and a 1-form such that, then the virtual contact structure of M is (ϕ, ξ, η).

$$\phi^2 = -I + \eta \otimes \xi, \quad \eta(\xi) = 1, \quad \eta \circ \phi = 0, \quad \phi\xi = 0.$$

For a more formal representation, we can write (M, ϕ, ξ, η). The premise fails unless g is a Riemannian metric on a nearly contact manifold M, as M is not a Riemannian manifold.

$$g(\phi X, \phi Y) = g(X,Y) - \eta(X)\eta(Y), \quad g(X,\xi) = \eta(X),$$
$$g(X, \phi Y) = -g(\phi X, Y),$$

In the notation (M, ϕ, ξ, η, g), we declare M to have the structure of a contact metric and specify vector fields X and Y on M M (ϕ, ξ, η, g).

The 1-form's exterior derivative is limited to the expressions (M, ϕ, ξ, η, g) under this condition.

$$d\eta(X,Y) = g(X, \phi Y),$$

Therefore, if (ϕ, ξ, η, g) is a contact metric structure, then the manifold M that contains (ϕ, ξ, η, g) is also a contact metric manifold. It is only when is a fatal vector field that M is also a

Riemannian manifold with K-contacts. Because of this, a Riemannian manifold with K-contacts is described by Sasakian [2] if the connection is Riemannian.

$$(\nabla_X \phi)Y = g(X,Y)\xi - \eta(Y)X$$

is right when the ∇ operator signifies the covariant differentiation of g. All these relationships are valid on a K-contact manifold M;

$$\begin{aligned} \nabla_X \xi &= -\phi X, \\ g(R(\xi,X)Y,\xi) &= g(X,Y) - \eta(X)\eta(Y), \\ R(\xi,X)\xi &= -X + \eta(X)\xi, \\ S(X,\xi) &= (n-1)\eta(X), \end{aligned}$$

Any time X, Y, and Z are all vector fields. , with R standing for M's Riemannian curvature and S for its Ricci tensor.

Definition: the manifold of which contains K-contacts Notably, due to the fact that M is locally symmetric,

$$\phi^2((\nabla_W R)(X,Y)Z) = 0,$$

The perpendicular is represented by the symbol ξ in the context of vector fields (X, Y, Z, and W).

Definition: Contact manifold with a K For M to be considered ϕ -symmetric, it must fulfill this requirement.

$$\phi^2((\nabla_W R)(X,Y)Z) = 0,$$

for unrestrained vector fields in X, Y, Z, and W

SEMI-SYMMETRIC NON-METRIC CONNECTION

M is assumed to be a Riemannian manifold of dimension n with a Riemannian metric g for the sake of clarity. If and only if M is a Levi-Civita linked Riemannian manifold, then the linear connection can be written as.

$$\overset{\circ}{\nabla}_X Y = \nabla_X Y + \eta(Y)X, \qquad (1)$$

The definition of the 1form related to the ξ on M vector field

$$\eta(X) = g(X,\xi), \qquad (2)$$

(see [1]). It is possible to compute the tensor of torsion T for the ◦∇ connection using the formula (1).

$$T(X,Y) = \overset{\circ}{\nabla}_X Y - \overset{\circ}{\nabla}_Y X - [X,Y] \quad (3)$$

satisfies

$$T(X,Y) = \eta(Y)X - \eta(X)Y. \quad (4)$$

Coordinating in a straight line ◦∇ When, it is claimed that the is semi-symmetric if and only if it satisfies (4). just in case these things hold true

$$\overset{\circ}{\nabla} g = 0.$$

It is maintained that ◦∇ if and only if ◦∇g 6= 0, then is a non-metric relationship. This is blatantly clear since (1).

$$(\overset{\circ}{\nabla}_X g)(Y,Z) = -\eta(Y)g(X,Z) - \eta(Z)g(X,Y) \quad (5)$$

in which X, Y, and Z on M are vector fields. Given definitions (4) and (5), it follows that is one of those non-metric semi-symmetric linkages. On a manifold M, the tensors R and R can be interpreted as representations of the Riemannian metric. Under these conditions, we can establish a connection between R and ◦R by

$$\overset{\circ}{R}(X,Y)Z = R(X,Y)Z - \alpha(Y,Z)X + \alpha(X,Z)Y \quad (6)$$

In this scenario, we have three tensor fields, X, Y, and Z, defined on a vector space, denoted by M. (0, 2).

$$\alpha(X,Y) = (\nabla_X \eta)Y - \eta(X)\eta(Y),$$

Research Methodology

A research technique is the standard procedure for collecting data, analysing it, and drawing findings. Research Methods refer to the procedures developed to carry out a study. Research, in a nutshell, is the act of systematically gathering and analyzing data for the objective of gaining new insights into a particular subject area. The primary purpose of this research is to apply scientific methods to actual issues. Much of the information presented in this research is secondary information gathered from open databases. The information used in this analysis was collected from a wide variety of online sources.

Result and Discussion

Theorem 1:

Possessing what we do,

(a) Any pseudo-Kaehler manifold M^n with $n = \dim_C M^n > 1$ is difficult to generate a non-trivial concircular vector field on.

(b) For pseudo-Kaehler manifolds Mn with n = 1, the result is invalid.

The following corollary follows naturally from the first theorem.

Corollary 1.1. Concurrent vector fields are those that are concircular on pseudo-Kaehler manifolds M^n, where n>1.

Proof of the theorem 1

On the complex n-dimensional pseudo-Kaehler manifold M^n, we allow any non-null concircular vector field v that satisfies a particular condition (1.1).

Now, we will characterize the Riemann curvature tensor R of M^n as

$$R(X,Y)Z = \nabla_X \nabla_Y Z - \nabla_Y \nabla_X Z - \nabla_{[X,Y]} Z$$

for fields with X, Y, and Z components that are M^n-tangent. The only tensor satisfying the above conditions is the Riemann curvature tensor R, thus that's the one you'll want to use.

$$R(X,Y) = -R(Y,X), \tag{2.2}$$

$$R(X,Y)JZ = J(R(X,Y)Z), \tag{2.3}$$

$$R(JX,JY)Z = R(X,Y)Z, \tag{2.4}$$

$$g(R(X,Y)Z,W) = g(R(Z,W)X,Y). \tag{2.5}$$

R is a tensor of curvature that agrees with

$$\begin{aligned} R(X,v)v &= \nabla_X \nabla_v v - \nabla_v \nabla_X v - \nabla_{[X,v]} v \\ &= \nabla_X(\mu v) - \nabla_v(\mu X) - \mu \nabla_X v + \mu \nabla_v X \\ &= (X\mu)v - (v\mu)X, \end{aligned} \tag{2.6}$$

Exactly if the vector space in M^n dimensions is tangent to the field X.In this case, we obtain by taking the inner product of v by (2.6).

$$(X\mu)g(v,v) = (v\mu)g(X,v), \quad \forall X \in TM^n. \tag{2.7}$$

Specifically, the following is implied by (2.7):

$$v\mu = 0 \ \text{ whenever } g(v,v) = 0, \tag{2.8}$$

$$X\mu = 0 \ \text{ whenever } g(v,v) \neq 0 \text{ and } g(X,v) = 0. \tag{2.9}$$

Let X denote the vector for which g(X, v) = 0. For X to be found, we need to take the inner product of (2.6), which yields:

$$g(R(X,v)v,X) = -(v\mu)g(X,X) \quad \text{whenever } g(X,v) = 0. \tag{2.10}$$

$$\begin{aligned} R(Y,Jv)Jv &= J(R(Y,Jv)v) \\ &= J\{\nabla_Y \nabla_{Jv} v - \nabla_{Jv} \nabla_Y v - \nabla_{[Y,Jv]} v\} \\ &= J\{\nabla_Y(\mu Jv) - \nabla_{Jv}(\mu Y) - \mu \nabla_Y(Jv) + \mu \nabla_{Jv} Y\} \\ &= -(Y\mu)v - ((Jv)\mu)JY \end{aligned} \tag{2.11}$$

only if the magnetic field M^n is perpendicular to the vector field Y.For example, if we plug in (2.11), we get; if we add (2.7), we get

$$g(R(Y, Jv)Jv, Y) = 0 \tag{2.12}$$

for any tangent vector Y satisfying $g(Y, v) = 0$.

Next, by applying (2.2), (2.4), (2.5) and (2.12) we have

$$\begin{aligned} 0 &= -g(R(Y, Jv)Jv, Y) = g(R(JY, v)Jv, Y) \\ &= g(R(Jv, Y)JY, v) = -g(R(v, JY)JY, v) \\ &= g(R(JY, v)v, JY) \end{aligned} \tag{2.13}$$

for any Y satisfying $g(Y, v) = 0$.

Now, by combining (2.10) and (2.13) we get

$$(v\mu)g(X, X) = 0 \tag{2.14}$$

for any tangent vector X satisfying $g(X, v) = g(JX, v) = 0$.

Proposition

However, vector fields cannot exist on pseudo-Kaehler manifolds with a non-zero (or positive) Ricci curvature. It follows as a logical extension of this line of thinking that.

Corollary: In the absence of a vector field, a Ricci-flat Einstein pseudo-Kaehler manifold is a sphere.

Remark: Using the pseudo-Kaehlerian metric C^n_s, we can do measurements in the non-Euclidean pseudo-Euclidean n-space.

$$g = -\sum_{i=1}^{s} dz_i d\bar{x}_i + \sum_{j=s+1}^{n} dz_j d\bar{z}_j,$$

possessing a Ricci flat pseudo-Kaehler manifold and a position vector field of C^n_s.

For any m > 2, N^m is thought to be a Riemannian m-manifold. For a piece of plane of shape T_pN^m, p N m, K(π) denotes the curvature. For any orthonormal basis $e_1, \ldots, e_m$ of T_pN^m, we give the following definition of the scalar curvature at p.

$$\tau(p) = \sum_{i<j} K(e_i \wedge e_j).$$

Assume that TpNm is a subspace of a set L, where r is an integer greater than 2, and that the set of orthonormal basis vectors {e1, . . . , er} is the set of orthonormal vectors in L. Scalar curvature of L, denoted by

$$\tau(L) = \sum_{\alpha<\beta} K(e_\alpha \wedge e_\beta), \quad 1 \le \alpha, \beta \le r.$$

S is the set of all possible m-tuples for some positive integer k ≥ 1. S(m, k) Collection (n_1,.., n_k) of integers where 2 ≤ n_1, · · · , n_k < m and $\sum_{j=1}^{k} i \leq m.$

In the early 1990s, the author defined the invariant (n_1, . . . , n_k) ∈ S(m, k) for each k-tuple of numbers δ(n_1, . . . , n_k) in

$$\delta(n_1, \ldots, n_k)(p) = \tau(p) - \inf\{\tau(L_1) + \cdots + \tau(L_k)\}, \;\; p \in N^m,$$

To specify all k orthogonal subspaces of T_pN^m, we use the notation dim $L_j = n_j$, j = 1, . . . , k, where L_1, . . . , L_k.

Assuming Kn is a Kaehlerian manifold, we can define X to be a holomorphically projective recurrent curvature killing vector field related to the vectorial field V.

$$L_X(\nabla_j P_{ki} - \nabla_k P_{ji}) = \{(n+2)R^a_{jki} - P_{ki}\delta^a_j + P_{ji}\delta^a_k - F^a_i H_{ki} + F^a_k H_{ji} + 2F^a_i H_{jk}\}V_{a.}(2.1)$$

Proof: Following the established commutation relation for tensors of type (0,2), we have

$$(L_X\nabla_j P_{ki} - L_X\nabla_k P_{ji}) - (\nabla_j L_X P_{ki} - \nabla_k L_X P_{ji}) = (L_X\Gamma^a_{ki})P_{ja} - (L_X\Gamma^a_{ji})P_{ka}(2.2)$$

However, in the spirit of hypothesis, if we take X to be a holomorphically projective curvature transformation, then we obtain by applying the following formula: (1.5).

$$L_X\Gamma^a_{ji} = \delta^a_j V_i + \delta^a_i V_j - F^a_j F^h_i V_h - F^a_i F^h_j V_h \ (2.3)$$

$$L_X P_{ji} = -(n+2)\nabla_j V_i \quad (2.4)$$

And a rationally, we obtain $L_X\Gamma^a_{ki}$ and $L_X P_{ki}$.

Through Substituting , we get

$$(L_X \nabla_j P_{ki} - L_X \nabla_k P_{ji}) - (\nabla_j[-(n + 2)\nabla_k V_i] - \nabla_k[-(n + 2)\nabla_j V_i])$$

$$= (\delta_k^a V_i + \delta_i^a V_k - F_k^a F_i^h V_h - F_i^a F_k^h V_h) P_{ja} - (\delta_j^a V_i + \delta_i^a V_j - F_j^a F_i^h V_h - F_i^a F_j^h V_h) P_{ka}$$

Finally, we conclude that through a few operations and some generalisation.

$$\{(n + 2)R_{jki}^a - P_{ki}\delta_j^a + P_{ji}\delta_k^a - F_i^a H_{ki} + F_k^a H_{ji} + 2F_i^a H_{jk}\}V_a = L_X(\nabla_j P_{ki} - \nabla_{ki} P_{ji})$$

Thereafter, several reports of earlier results would be submitted, and

A.If $\nabla_j P_{ki} = \nabla_k P_{ji}$ then K_n is Kaehler-Peterson-Codazzi manifolds and $\{(n + 2)R_{jki}^a - P_{ki}\delta_j^a + P_{ji}\delta_k^a - F_i^a H_{ki} + F_k^a H_{ji} + 2F_i^a H_{jk}\}V_a = 0.$(2.5)

The fact that a Kaehler-Peterson-Codazzi manifold is harmonically curvy is, therefore, of paramount importance.

$$\nabla_j P_{ki} = \nabla_k P_{ji} \Leftrightarrow \nabla_a R_{jki}^a = 0.$$

Additionally, Kaehler-Peterson-Codazzi manifolds exist only when the manifold's scalar curvature is constant, making them a special case of Einstein manifolds. Multiplying g^{ki} by gives us a more accurate result of (2.5)

$$\{(n + 2)g^{ki} R_{jki}^a - R\,\delta_j^a + g^{ai} P_{ji} - F_i^a g^{ki} H_{ki} + F g^{ki} H_{ji} + 2F_i^a g^{ki} H_{jk}\}V_a = 0.$$

While $V_a \neq 0$ and budding the three previous terms we comprise

$$(n+2)g^{ki}R^{a}_{jki} - R\delta^{a}_{j} + g^{ai}P_{ji} - F^{a}_{i}F^{b}_{k}g^{ki}P_{bi} + 3F^{a}_{k}F^{b}_{j}g^{ki}P_{bi} = 0,$$

Next to creation the reduction α = F andestimate starting 1 to n then we get holdof:

$$g_{ki}(nR + 2R - nR + R)3\,P_{ki,}$$

Hence like this, we complete that $P_{ki} = \frac{R}{n} g_{ki}.$ then further expressions theKaehler- Peterson-Codazzi manifold is anEinsteinian manifold.

References

1. Ozg ¨ ur, C. and Sular, S. Warped products with semi-symmetric non-metric connections. ¨ Arab. J. Sci. Eng., 2011, 36, 461–473.
2. Sular, S. and Ozg ¨ ur, C. Warped product manifolds with semi-symmetric metric connections. ¨ Taiwan. J. Math., 2011, 15,1701–1719.
3. C.S. Bagewadi, D.G. Prakasha and Venkatesha, Projective curvature tensor on a Kenmotsu manifold with respect to semi-symmetric metric connection, Stud. Cerc. St. Ser. Mat. Univ. Bacau. 17 (2007), 21-32.
4. D.G. Prakasha, C.S. Bagewadi and Venkatesha, Conformally and quasicon formally conservative curvature tensors on a trans-Sasakian manifold with respect to semi-symmetric metric connections, Diff. Geom. Dyn. Syst. 10 (2008),263-274
5. B.-Y. Chen, A simple characterization of generalized Robertson-Walker space times, Gen. Relativity Gravitation 46 (2014), no. 12, Art. 1833, 5 pp.
6. B.-Y. Chen, Ricci solitons on Riemannian sub manifolds, Riemannian Geometry and Applications (2014), 30–45.
7. B.-Y. Chen, Total Mean Curvature and Sub manifolds of Finite Type, second Edition, World Scientific, 2015.
8. B.-Y. Chen, Some results on concircular vector fields and their applications to Ricci solitons, Bull. Korean Math. Soc. 52 (2015), no. 5, 1535-1547.
9. Malave Guzman. Transformation esholomorficamemte projective as equivalentes, Department Mathematic dela Universidad de oriente, (tesis demaestria) 2007
10. U.S. Negi. Some problems concerning Pseudo-analytic vectors on Pseudo-Kaehlerian Manifolds. AARJMD, vol. 1,issue 17, pp. 106-113, 2014.
11. Lukas Rachunek, Josef Mikes, On tensor Fields semi conjugated with Torse-Forming vector fields, Acta Univ.Palacki.Olomuc., Fac.rer.nat.,Mathematica., 44 (2005), 151-160.
12. C. S. Bagewadi and Venkatesha, Torseforming Vector field in a 3 dimensional trans-Sasakian manifold, Differential Geometry. Dynamical Systems. 8 (2006), 23-28.

13. C. S. Bagewadi, D .G. Prakasha and Venkatesha, Torseforming Vector field in a 3-dimensional contact metric manifold,. General Mathematics, 16(1), (2008), 83-91
14. P. Alegre and A. Carriazo, Structures on generalized Sasakian -Space-forms, Differential Geom.and its Appl. 26 (2008), 656-666.
15. P.Alegre, D. Blair and A. Carriazo, Generalized Sasakian-Space-forms, Israle J.Math. 141(2004), 157-183